Macroeconomics I
Homework 3

This homework is due on December the 3^{rd} at 5 p.m.

Exercise 1

In this exercise you need to solve the social planner's problem of Neoclassical Growth Model with uncertainty and labor/leisure choice. As we saw in class the problem can be written as:

$$V(k,z) = \max_{k',l}\Big\{u(c,l) + E_{z'|z}[V(k',z')]\Big\}$$
$$\text{s.t. } c + k' = f(k,l) + (1-\delta)k$$
$$z' = \rho z + \epsilon, \epsilon \sim N(0,\sigma_\epsilon^2)$$
$$k' \in \Gamma(k,z)$$
$$l \in [0,1]$$

we are going to assume that $f(z,k,l) = e^z k^\alpha l^{1-\alpha}$ with $\alpha = 0.36$, $\beta = 0.96$, $\delta = 0.025$, $\rho = 0.95$, $\sigma_\epsilon = 0.007$ and utility function:

$$u(c,l) = (1-\theta)\log c + \theta log(1-l),$$

with $\theta = 0.64$

a Create a tensor which stores the optimal fraction of time working versus leisure conditional on each possible combination of (k,k',z) : $l^*(k,k',z)$.

b Create a grid of capital around the steady state level of capital in a world without uncertainty and exogenous labor supply with $nkk = 400$ points in the grid. You can start your grid in $k_1 = 0.5k^*$.

c Create an equally spaced grid of productivity shocks and create the transition probability matrix using the Tauchen method that we saw in class. Set the number of points in the grid of shocks equal to $nzz = 12$ and go 2 standard deviations away from 0.

d Fill in a $nkk \times nkk \times nzz$ tensor containing the utility level associated with any feasible combination of (k,z,k') using the optimal labor choice $u^*(k,z,k') = u(f(k,l^*) + (1-\delta)k - k')$

e Create a initial guess of your value function V^g

   (a) Create a matrix that contains $EV(k',z')$.
   (b) Loop over k, z and k' for finding the optimal k'. (It would be good to economize in at least one of these loops but if you can't, don't worry too much; your code will take a bit longer but doable.)
   (c) Create matrix containing the new V^{g+1}.
   (d) Compute the maximum distance between these two functions: $\sup(\parallel V^{g+1} - V^g \parallel)$
   (e) If this distance is greater than an ϵ go back to a) setting $V^g = V^{g+1}$

f Create a matrix containing the policy function for capital and labor

- g Simulate the path of capital for your economy for 50000 periods using the policy function and report the stationary distribution of capital (another more advanced way to get at the distribution of capital would be to use the policy function and determine the transition probability across possible states of the world which is a transition matrix of dimension $(nkk \times nzz) \times (nkk \times nzz)$. The stationary distribution is the eigenvector associated with the unit eigenvalue.)

- h You should report:
  - (a) Number of iteration over the value function until convergence and computing time.
  - (b) Plot of your final value function $V(k,1), V(k,6), V(k,12)$ as a function of current capital.
  - (c) Plot the policy function for capital and labor for these 3 same shocks as a function of capital.
  - (d) Histogram for the stationary distribution of capital.

Exercise 2

Consider the following search problem. An unemployed agent receives one wage offer each period. There are two possible wage offers: w_1 and w_2 with $w_2 > w_1 > 0$. Suppose the probability of receiving w_1 is π and the probability of receiving w_2 is $(1-\pi)$. If the worker accepts an offer, he receive that wage as his income forever. If the worker rejects an offer, his current income is $b > 0$ and he receives another offer next period. The agent discounts the future at rate $\beta \in (0,1)$ and maximizes the expected present discounted value of his income. Let $v(w_1)$ and $v(w_2)$ be the value functions of an unemployed agent when he/she has w_1 and w_2 as current wage offers, respectively.

- a Define $v(w_1)$ and $v(w_2)$

- b Solve for the agent's optimal decision.