Consumption and Saving

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Consumption and Saving

- Every day, people face a basic choice:
  - consume today
  - or save for the future
- Examples:
  - spend all your salary or save part of it?
  - buy a new phone now or put money aside?
- This chapter asks:
  - How do people decide how much to consume?
  - Why do people save?
  - How do income and interest rates affect these decisions?

A Simple Two-Period World

- Life is divided into two periods:
  - Period 1: today
  - Period 2: tomorrow
- You receive income in each period
  - y_1 today
  - y_2 tomorrow
- You choose how much to:
  - consume today (c_1)
  - consume tomorrow (c_2)

A Savings Technology

- You have access to a simple way to save money
  - think of a bank account
- In period 1, you can save part of your income y_1
  - let s be the amount you save
- In period 2, your savings grow
  - saving s today gives $(1 + R)s$ tomorrow
- R is the **interest rate**

Saving and Borrowing

- Let s be savings in period 1
  - $s > 0$: savings
  - $s < 0$: borrowing
- In period 2:
  - saving or borrowing changes resources by $(1 + R)s$
- Same interest rate for both

The Consumption–Saving Model

- Budget constraints in each period:
- Period 1 (today):

$$c_1 + s = y_1$$

- Period 2 (tomorrow):

$$c_2 = y_2 + (1 + R)s$$

Limits on Saving and Borrowing

- How much can you save?
  - you cannot save more than your income today

$$s \leq y_1$$

- How much can you borrow?
  - you must be able to repay tomorrow

$$(1 + R)s \geq -y_2$$

which implies:

$$s \geq -\frac{y_2}{1 + R}$$

Utility Function

- Preferences describe how people value consumption
- In general, utility depends on consumption in both periods:

$$W(c_1, c_2)$$

- We will use a simple and common specification:

$$W(c_1, c_2) = U(c_1) + \beta\, U(c_2)$$

- Utility today plus discounted utility tomorrow

Interpretation of the Discount Factor

- β is the **subjective discount factor**
- It measures how much weight you place on future consumption
- Interpretation:
  - Higher β: more patient
  - Lower β: more impatient
- If $\beta < 1$:
  - future consumption is discounted
- If one period is one year, a typical value is:

$$\beta \approx 0.98$$

The Consumption–Saving Model

- The student chooses consumption to maximize:

$$\max_{c_1, c_2, s} U(c_1) + \beta\, U(c_2)$$

- Subject to:

$$c_1 + s = y_1$$

$$c_2 = y_2 + (1 + R)s$$

- And limits on saving and borrowing

Solving the Model: Step 1

- Combine the two budget constraints into one
- Intertemporal budget constraint:

$$c_1 + \frac{1}{1+R} c_2 = y_1 + \frac{1}{1+R} y_2$$

- This eliminates saving s from the problem
- Interpretation:
  - present value of consumption
  - equals present value of income

Present Value

- Question:
  - how much does it cost **today** to get 1 unit of consumption tomorrow?

- If you save 1 unit today, tomorrow you get:

$$1 + R$$

- Therefore, to get 1 unit tomorrow, you must save today:

$$\frac{1}{1 + R}$$

- This is the **present value** of one unit of future consumption

Understanding Present Value

- Imagine you want to buy a surfboard next year for €500
- How much do you need to save today if the bank pays interest R?

$$\text{Today's cost} = \frac{500}{1+R}$$

- Example:
  - If $R = 0.05$ (5% interest)
  - To get €500 next year, you save:

$$\frac{500}{1.05} \approx 476.19 \text{ euros today}$$

- **Intuition:** *If you want to buy that surfboard next year, you only need €476 today if you put it in the bank at 5%*
- Present value computes the price today of future consumption: Buying a surf board for period 2 is cheaper than buying it today.

Solving the Model: Step 2

- Use the intertemporal budget constraint to eliminate c_2
- From the budget constraint:

$$c_2 = (1+R)\left(y_1 + \frac{y_2}{1+R} - c_1\right)$$

- Plug into utility:

$$U(c_1) + \beta U(c_2)$$

- Now the problem has:
  - one choice variable: c_1

Solving the Model: Step 3

- The household chooses c_1 to maximize:

$$U(c_1) + \beta U\left((1+R)\left[y_1 + \frac{y_2}{1+R} - c_1\right]\right)$$

- Take the derivative with respect to c_1
- Set it equal to zero (first-order condition)

The Consumption Euler Equation

- First-order condition:

$$U'(c_1) = \beta(1 + R)U'(c_2)$$

- This is called the **Euler equation**
- Interpretation:
  - marginal utility today
  - equals discounted, interest-adjusted marginal utility tomorrow

Euler Equation: Variational Argument

- Start from a candidate optimal choice (c_1, c_2)
- Suppose the student considers saving a tiny bit more today
- Marginal cost (today):

$$U'(c_1)$$

- Marginal benefit (tomorrow):

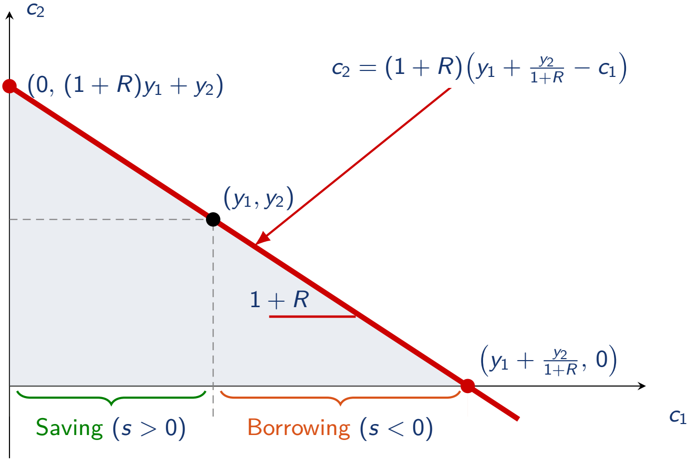
$$\beta(1+R)U'(c_2)$$

- Optimal choice requires:

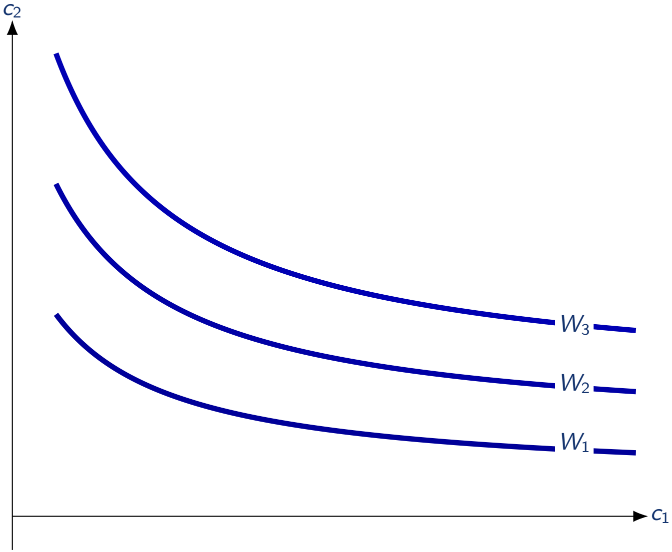
$$U'(c_1) = \beta(1+R)U'(c_2)$$

- This is the **consumption Euler equation**

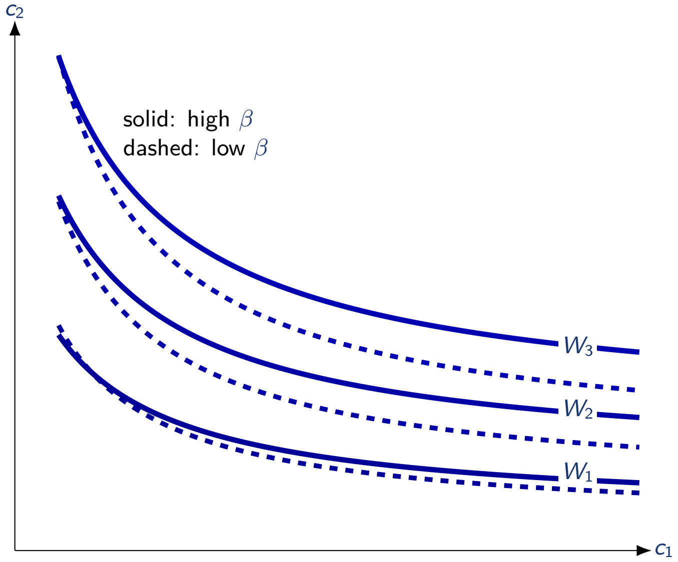
The Budget Constraint



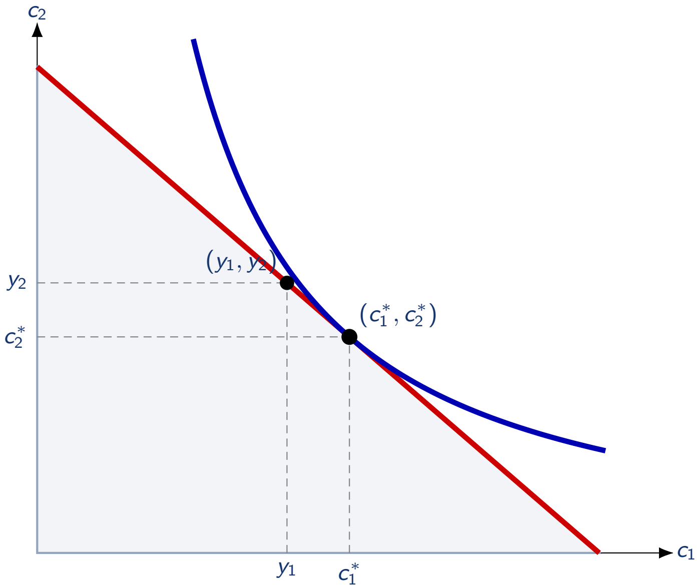
Preferences over Consumption



Preferences over Consumption: More Impatient Household



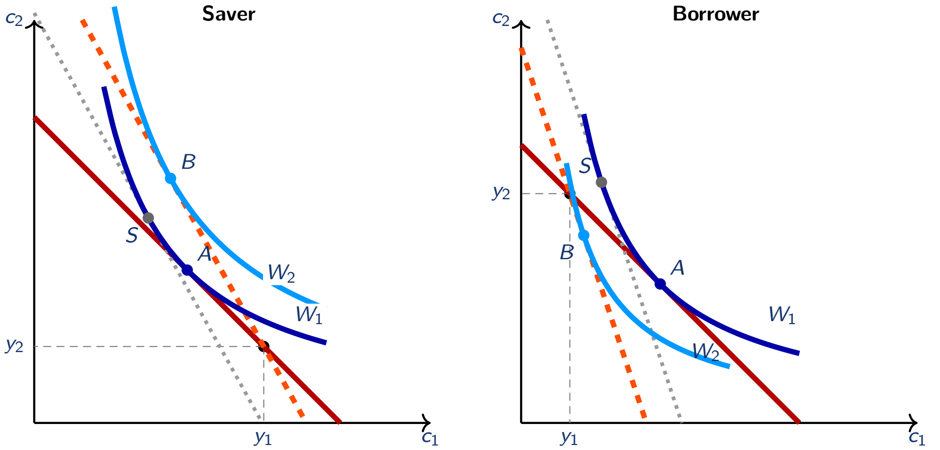
Optimal Choice



An Increase in the Interest Rate

- When R increases, the budget line becomes **steeper** around the endowment point (y_1, y_2)
- **Substitution effect:**
  - future consumption becomes cheaper relative to current consumption
  - households want to shift consumption from period 1 to period 2
  - therefore: lower c_1, higher c_2
- **Income effect:** depends on whether the household is a saver or a borrower
  - if the household is a **saver** $(s > 0)$, higher R raises lifetime resources
  - if the household is a **borrower** $(s < 0)$, higher R lowers lifetime resources
- Therefore:
  - **Saver:** income effect raises both c_1 and c_2
  - **Borrower:** income effect lowers both c_1 and c_2
  - total effect on c_1 is ambiguous for savers, while total effect on c_2 is ambiguous for borrowers

Higher Interest Rate: Saver vs Borrower



Euler Equation with Log Utility

- Assume:

$$U(c) = \ell\text{og}\, c \quad \Rightarrow \quad U'(c) = \frac{1}{c}$$

- Total utility is given by:

$$W(c_1, c_2) = U(c_1) + \beta U(c_2) = \ell\text{og}(c_1) + \beta\, \ell\text{og}(c_2)$$

- The max of W is found when Euler equation holds:

$$\frac{1}{c_1} = \frac{\beta(1+R)}{c_2} \quad \Rightarrow \quad c_2 = \beta(1+R)c_1$$

- The Euler equation implies that consumption grows by factor $\beta(1+R)$

Consumption Growth

- With log utility:

$$\frac{c_2}{c_1} = \beta(1 + R)$$

- Notice:
  - Consumption growth does NOT depend on income growth: y_1/y_2
  - Whether you earn most of your income in period 1 or period 2, consumption grows at the same rate

Consumption-Savings: Why Not Just Track Income?

- Suppose most of your income comes in period 2
- You could let consumption follow income:

$$c_1 = y_1, \quad c_2 = y_2$$

- But this is not optimal because of diminishing marginal utility:
  - Marginal utility is higher in period 1 (less consumption)
  - Marginal utility is lower in period 2 (lots of consumption)
- By transferring some consumption to period 1, total utility increases

Consumption Smoothing

- Assume:

$$\beta(1+R)=1$$

- Then the Euler equation implies:

$$c_1 = c_2$$

- Interpretation:
  - Optimal consumption does not depend on the timing of income
  - Even if y_1 is small and y_2 is large, consumption is smoothed
  - This is because of diminishing marginal utility

Consumption Growth and the Interest Rate

- Euler equation (log utility):

$$\frac{c_2}{c_1} = \beta(1 + R)$$

- Holding β fixed:
  - Higher $R \rightarrow$ higher growth rate of consumption
  - Why? Higher return on saving makes future consumption cheaper
  - You save more today $\rightarrow$ consumes less today, more tomorrow

Solving for Consumption Levels

- With log utility, we have two equations:

$$\frac{c_2}{c_1} = \beta(1+R)$$
$$c_1 + \frac{1}{1+R}c_2 = y_1 + \frac{1}{1+R}y_2$$

- Unknowns: c_1 and c_2
- Solve these two equations to get the level of consumption in each period

Level of Consumption: Solution

- Solving the system yields:

$$c_1 = \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+R}\right)$$

- Consumption at time 1 is a fraction of the present value of lifetime income
- Current income only matters insofar as it affects the present value
- Current income is not special: consumption smooths over both periods
- Permanent income hypothesis: what matters is life-time income and not any one period's income.
- Two individuals A and B with $y_1^A \neq y_1^B$ and $y_2^A \neq y_2^B$ but with $y_1^A + \frac{y_2^A}{1+R} = y_1^B + \frac{y_2^B}{1+R}$ should consume the same

Marginal Propensity to Consume (MPC)

Suppose you receive a surprise scholarship of **€1000**.

Question: How much would you spend within a year, and how much would you save for later?

Marginal Propensity to Consume (MPC)

Suppose you receive a surprise scholarship of **€1000**.

Question: How much would you spend within a year, and how much would you save for later?

- Think about whether you expect this scholarship to be a one-time bonus or a permanent increase in your income.
- How does that affect your decision to spend now versus later?

Marginal Propensity to Consume out of transitory shock

- Suppose the agent income y_1 increases by a small amount, but the agent does **not** change its expectation of future income y_2
- This is perceived as a **transitory increase** (e.g., a one-time bonus).
- How much does consumption respond? The answer is given by the **marginal propensity to consume** to a transitory shock
- In our example with log utility:

$$\text{MPC}_t = \frac{\partial c_1}{\partial y_1} = \frac{1}{1+\beta} < 1$$

- The more impatient you are, the more you consume out of a transitory income shock

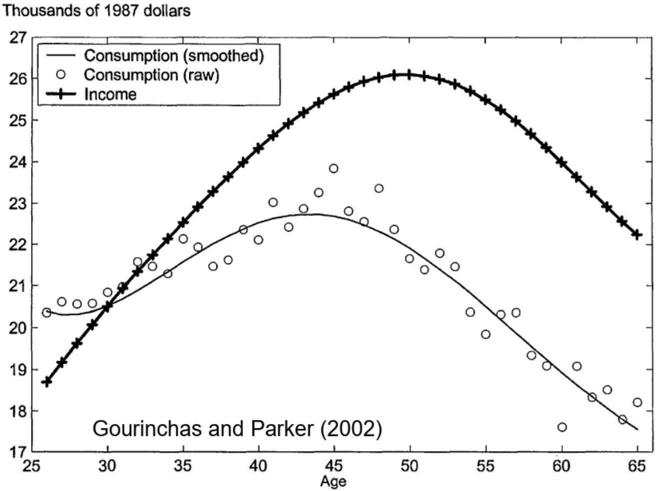
Marginal Propensity to Consume out of permanent shock

- Suppose the agent income y_1 and y_2 increases by a small amount Δ
- This is perceived as a **permanent increase** (e.g., a promotion).
- How much does consumption respond? The answer is given by the **marginal propensity to consume** to a permanent shock
- In our example with log utility:

$$\text{MPC}_p = \frac{\partial c_1}{\partial \Delta} = \frac{1}{1+\beta} + \frac{1}{1+\beta}\frac{1}{1+R} > \frac{1}{1+\beta}$$

- In the case $\beta(1+R) = 1$ the $MPC_p = 1$: your consumption increases one for one with your income.

Does the Model fit the data?



Does the Model fit the data?

- **Answer:** No
- The model implies a **constant growth rate of consumption** (from the Euler equation)
- Empirical data show a **hump-shaped pattern of consumption** over the life cycle
- Observed consumption profiles **track income patterns**, whereas the model predicts consumption should be smooth
- This highlights the need for more realistic features: uncertainty, borrowing constraints, and life-cycle considerations.
- $\beta(1+R) < 1$ and credit constraints can replicate the pattern

Key Takeaways: Consumption-Saving

- **Consumption Smoothing:**
  - Diminishing marginal utility implies households should **borrow when income is low** and **save when income is high**.
- **Borrowing Constraints:**
  - Limits on borrowing prevent households from smoothing consumption as much as they would like.