

Consumption and Saving

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Exercise 1. From the Budget Constraint to the Optimal Choice

Consider a two-period household with income

$$y_1 = 4, \quad y_2 = 8,$$

interest rate

$$R = 0.25,$$

and preferences

$$W(c_1, c_2) = \log(c_1) + \beta \log(c_2), \quad \beta = 0.8.$$

(a) Write the budget constraint in each period:

$$c_1 + s = y_1, \quad c_2 = y_2 + (1 + R)s.$$

Then combine the two equations into one intertemporal budget constraint relating c_1 and c_2 only.

(b) Compute:

- the endowment point (y_1, y_2) ,
- the horizontal intercept of the budget line,
- the vertical intercept of the budget line.

Explain which side of the endowment point corresponds to saving and which side corresponds to borrowing.

(c) Write the Euler equation for this problem and show that optimal consumption must satisfy

$$\frac{c_2}{c_1} = \beta(1 + R).$$

(d) Using the numerical values of β and R , compute the growth rate of consumption:

$$\frac{c_2}{c_1}.$$

Is consumption rising, falling, or constant over time?

(e) Consider the following two feasible consumption allocations:

$$A = (c_1, c_2) = (3, 9.25), \quad B = (c_1, c_2) = (5, 6.75).$$

For each allocation:

- compute total utility $W(c_1, c_2)$,
- compute $U'(c_1)$ and $\beta(1 + R)U'(c_2)$,
- check whether

$$U'(c_1) > \beta(1 + R)U'(c_2) \quad \text{or} \quad U'(c_1) < \beta(1 + R)U'(c_2),$$

- explain whether the household should increase or decrease c_1 .

In words, explain the economic meaning of each inequality.

(f) Use the Euler equation together with the intertemporal budget constraint to solve for optimal c_1 and c_2 .

(g) Compute optimal saving

$$s = y_1 - c_1.$$

Is the household a saver or a borrower?

Exercise 2. How Does Preferences Change Consumption Growth?

Keep the same income process and interest rate as in Exercise 1:

$$y_1 = 4, \quad y_2 = 8, \quad R = 0.25, \quad \beta = 0.8.$$

But now suppose preferences are

$$W(c_1, c_2) = \sqrt{c_1} + \beta\sqrt{c_2}.$$

- Compute marginal utility in each period.
- Write the Euler equation for this problem.
- Using the numerical values of β and R , compute the ratio c_2/c_1 . Compare it with the log-utility case from Exercise 1.
- Use the intertemporal budget constraint to solve for optimal c_1 and c_2 under this utility function.

(e) Compute saving

$$s = y_1 - c_1.$$

Is the household saving or borrowing?

(f) Suppose the interest rate rises from $R = 0.25$ to $R = 0.50$, while income and β stay unchanged.

- Under log utility, what happens to the ratio c_2/c_1 ?

- Under \sqrt{c} utility, what happens to the ratio c_2/c_1 ?
- Which utility function makes consumption growth more sensitive to the interest rate?

Exercise 3. Temporary versus Persistent Income Shocks

Suppose a household has log utility and solves the two-period consumption-saving problem that we saw in class. Assume:

$$\beta = 0.8, \quad R = 0.25.$$

(a) Start from the baseline income profile

$$y_1 = 4, \quad y_2 = 4.$$

Compute optimal current consumption c_1 .

(b) **One-time bonus.** Suppose the household receives an extra 2 units of income today only, so income becomes

$$y_1 = 6, \quad y_2 = 4.$$

Compute the new optimal c_1 . How much of the bonus is consumed today, and how much is saved?

(c) **More persistent increase.** Instead, suppose income rises by 1 unit in each period, so income becomes

$$y_1 = 5, \quad y_2 = 5.$$

Compute the new optimal c_1 . Compare the increase in current consumption with part (b).

(d) Which shock leads to a bigger increase in current consumption: the one-time bonus in part (b) or the more persistent increase in part (c)? Explain in terms of lifetime resources rather than current income alone.

(e) Consider two alternative shocks:

- Shock T: y_1 rises by 1.8 and y_2 does not change.
- Shock P: both y_1 and y_2 rise by x .

Find the value of x such that the two shocks have the same present value. Then compare their effect on optimal current consumption.