Chapter III: Economic Growth

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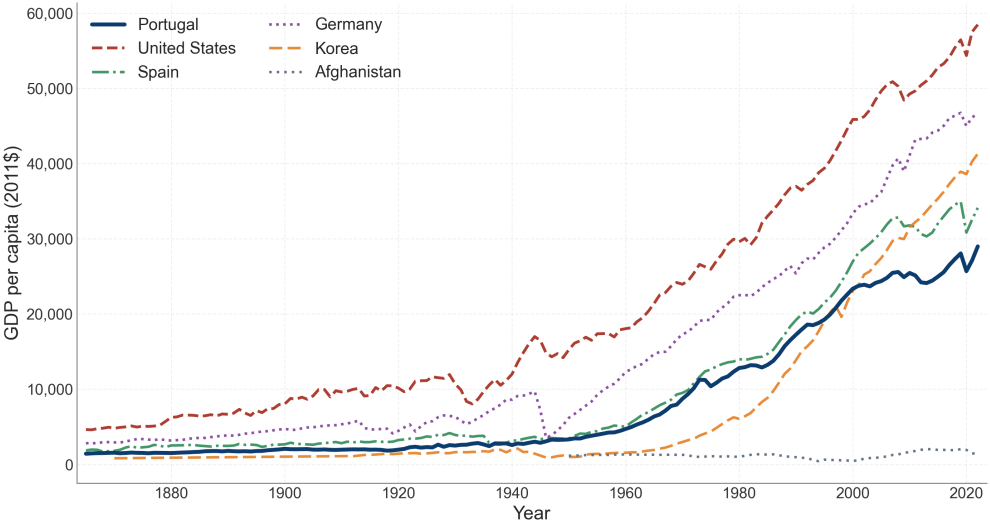
Why Study Economic Growth?

- As we saw in chapter I, GDP shapes to a large extent living standards.
- Even **small differences** in gdp growth rates create **huge income gaps** over time.
- Rule of thumb: if gdp grows at $g\%$ per year,

$$\text{Doubling time} \approx \frac{70}{g} \text{ years.}$$

- Examples:
  - **South Korea vs. Afghanistan**: similar income per capita in the 1960s, but sustained faster growth made Korea one of the richest countries in Asia while Afghanistan became x10 poorer.
  - **Poland vs. Portugal**: Portugal was x3 richer in GDP per capita in the early 1990s, yet Poland's consistently higher growth has allowed it to catch up and even surpass Portugal.
- Understanding what drives growth helps us understand why some countries prosper while others fall behind.

GDP per Capita Since 1865



Source: Maddison Project Database 2023 (Bolt and van Zanden et al.).

The Kaldor Facts

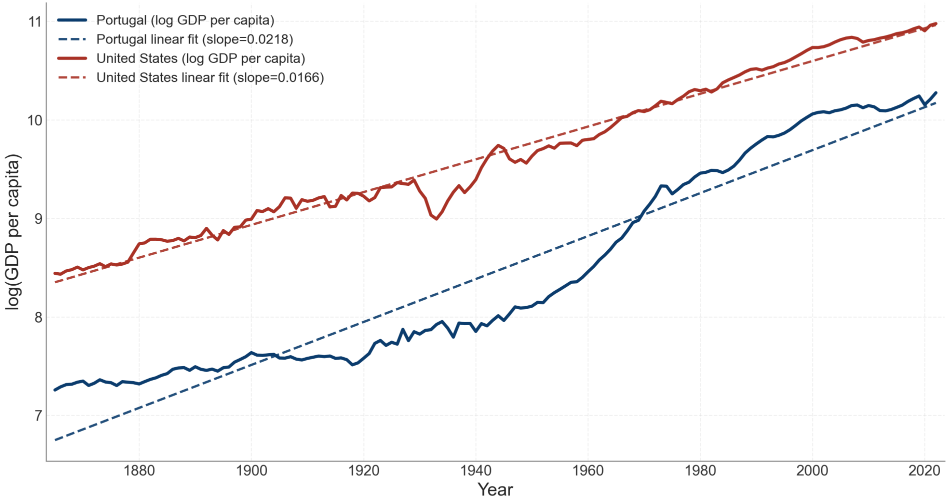
- In 1957, Nicholas Kaldor highlighted a set of regular patterns in growing economies.
- He called them **"remarkable historical constancies"**; today we refer to them as the **Kaldor Facts**.
- These facts are useful because they tell us what any good growth model should be able to explain.
- Our question today: do these patterns still hold?

The Kaldor Facts

Fact 1 The rate of growth of GDP per capita is constant

- A straight line (in log scale) seems to do quite well in describing how the US economy has grown for many decades and a bit less so for Portugal.
- Constant growth rates appear as straight lines in logs.
- GDP per capita has been growing at a rate of approximately 1.5% and 2.0% per year for a long time for the US and Portugal respectively.
- Again, notice that while 2% doesn't seem like a lot, compounded over time it amounts to a huge increase in GDP per capita.
- It implies that the **doubling time** of GDP per capita is around 35 years for the US and 30 years for Portugal.

Log GDP per Capita Since 1865: Portugal vs United States



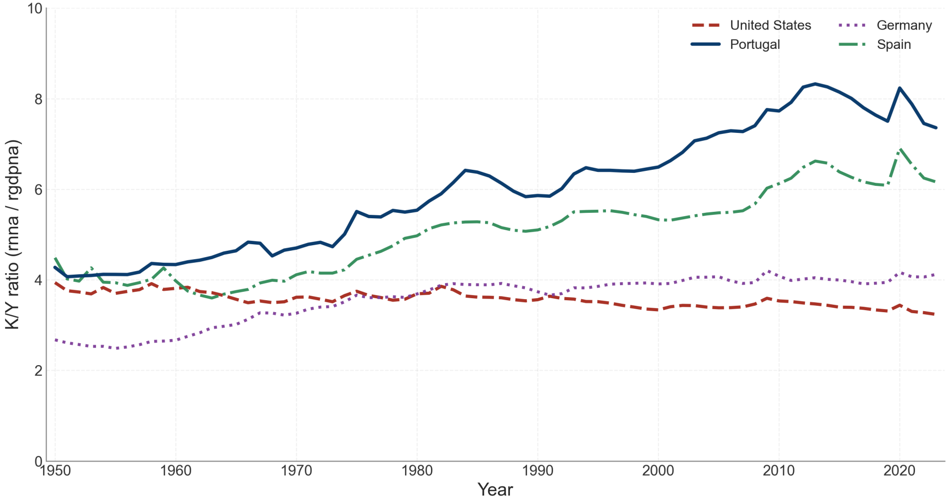
Source: Maddison Project Database 2023 (Bolt and van Zanden et al.), GDPpc sheet.

The Kaldor Facts

Fact 2 The ratio of the total capital stock to GDP is constant

- The capital stock includes the value of all machines, buildings, and equipment used in production.
- It is typically measured by cumulating past investment and subtracting depreciation.
- In the U.S., this ratio has been stable at about **3.5**: the economy holds capital worth about 3.5 years of GDP.

Kaldor Fact 2: K/Y



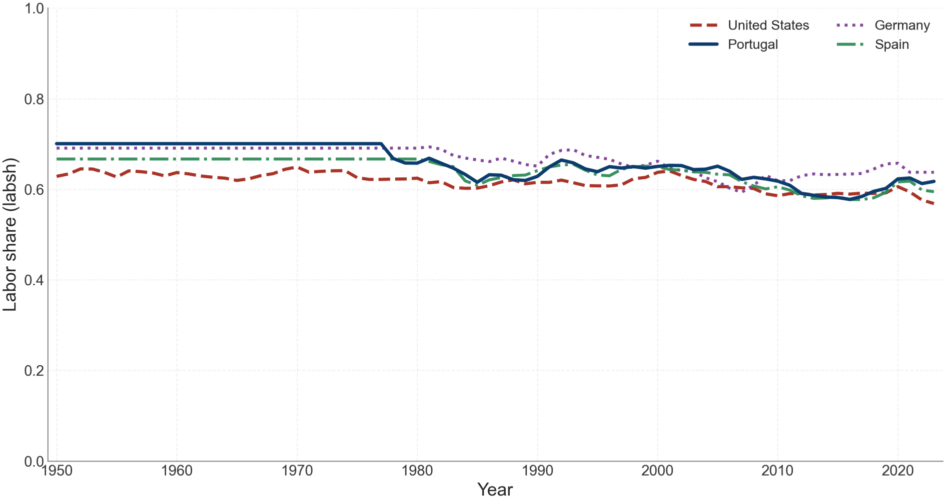
Source: Penn World Table 11.0 (pwt110.dta). Variables: rnna and rgdpna.

The Kaldor Facts

Fact 3 The shares of labor and capital income in GDP are constant

- Recall from Chapter 1 the income method of measuring GDP
- GDP can be split into labor income (wages) and capital income (profits, rents).
- Some income categories (e.g. proprietors' income) are hard to classify; a common approach is to exclude them and thus assume equal factor shares than in the rest of the economy.
- Historically, the labor share of GDP was remarkably stable at about 65%.
- Since around 2000, labor's share has declined by roughly 3 percentage points.

Kaldor Fact 4: Stability of Labor Share



Source: Penn World Table 11.0 (pwt110.dta). Variable: labsh.

Kaldor Facts

Fact 4 Constant Rate of Return on Capital

- The rate of return on capital measures income earned per unit of capital
- Empirically, this return has remained roughly constant over time.
- It's not a separate fact since it's implied by facts 2 and 3. Why?
  - Fact 2: capital stock relative to GDP is stable.
  - Fact 3: capital's share of income is stable.

$$r = \frac{\text{Capital Income}}{K} = \frac{\text{Capital Income}/Y}{K/Y}$$

Why Do We Need the Solow Model?

The Kaldor Facts tell us *what*. . .

- GDP per capita grows at a constant rate
- Capital-output ratio is stable
- Factor shares are roughly constant

The Solow Model (1956) asks *why*

- Why are some countries richer than others?
- Can poorer countries catch up?
- What limits long-run growth?

. . . but *not* **why**.

- Sustained per-capita growth requires **technological progress** — savings and capital accumulation alone are not enough.
- Simple, tractable, and surprisingly powerful: the **workhorse model** of economic growth.
- Next: build the model and see which Kaldor facts it can—and cannot—match.

The Solow–Swan Model

- The Solow–Swan (1956) model is a simple model to study long-run economic growth.
- It provides a description of how capital accumulation and technology shape GDP per capita.
- In contrast to the model that we saw in chapter 2, the capital stock is going to be determined within the model.
- Let's see the main ingredients.

Production Function

- A production function tells us how inputs become output:

$$Y = F(K, L)$$

- Inputs:
  - K: capital (machines, buildings, equipment)
  - L: labor (hours of work)

Consumption and Investment

- The economy is closed and there is no government and population is constant: $\bar{L}$
- The national income identity simplifies to:

$$Y = C + I$$

- All output is either consumed or invested.

Savings and Investment

- The savings rate is exogenous and constant:

$$\frac{S}{Y} = s.$$

- Income is equal to output: Y is both.
- Savings: $S = Y - C$.
- Savings rate: $\frac{Y-C}{Y} = s$.
- In a closed economy, savings equal investment: $S = I$.

Depreciation and Capital Accumulation

- In contrast to the previous chapter, this model is going to be dynamic.
- The capital stock depreciates at a constant rate δ.
- Capital wears out: machines break, buildings deteriorate, technology becomes obsolete.
- Each period, a fraction δ of the capital stock loses its productive value.

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- $(1 - \delta)K_t$: capital that survives depreciation.
- I_t: new capital created through investment.

Capital Accumulation: A Numerical Example

- Consider an economy with initial capital $K_0 = 1,000$ units, constant investment $I_t = 200$ units, and depreciation rate $\delta = 0.10$.
- How does the capital stock evolve?

Time t	Capital K_t	Investment I_t	Depreciation δK_t	Change ΔK_{t+1}
0	1,000	200	100	100
1	1,100	200	110	90
2	1,190	200	119	81
3	1,271	200	127	73
4	1,344	200	134	66
5	1,410	200	141	59

- Each period: $\Delta K_{t+1} = I_t - \delta K_t$.
- Net investment decreases over time as depreciation rises with the capital stock.

Per Capita Production Function

We rewrite the production function in per-capita terms. Define:

$$y \equiv \frac{Y}{\bar{L}}, \qquad k \equiv \frac{K}{\bar{L}}$$

where y is GDP per capita and k is capital per worker.

Starting from the aggregate production function:

$$y = \frac{F(K, \bar{L})}{\bar{L}} = F\left(\frac{K}{\bar{L}}, 1\right) \equiv f(k)$$

- The first step substitutes $Y = F(K, \bar{L})$ from the aggregate production function.
- Because of constant returns to scale, scaling both inputs by $1/\bar{L}$ leaves output per worker well-defined.
- We define the per-capita production function $f(k)$ as the output produced by one worker with k units of capital.

Dynamics of Capital per Worker

We want to understand how capital per worker evolves over time:

$$k_{t+1} \equiv \frac{K_{t+1}}{\bar{L}}$$

Start from the capital accumulation equation:

$$K_{t+1} = (1-\delta)K_t + I_t$$

Use:

$$I_t = sY_t$$

Combining these:

$$k_{t+1} = (1-\delta)k_t + sy_t$$

$$\Rightarrow \Delta k_{t+1} = (1-\delta)k_t + sy_t - k_t = sf(k_t) - \delta k_t$$

Interpretation of the Dynamics

The evolution of capital per worker depends on two opposing forces:

- **Investment per worker:**

$$\underbrace{sf(k)}_{\text{adds to } k}$$

- **Depreciation:**

$$\underbrace{\delta k}_{\text{subtracts from } k}$$

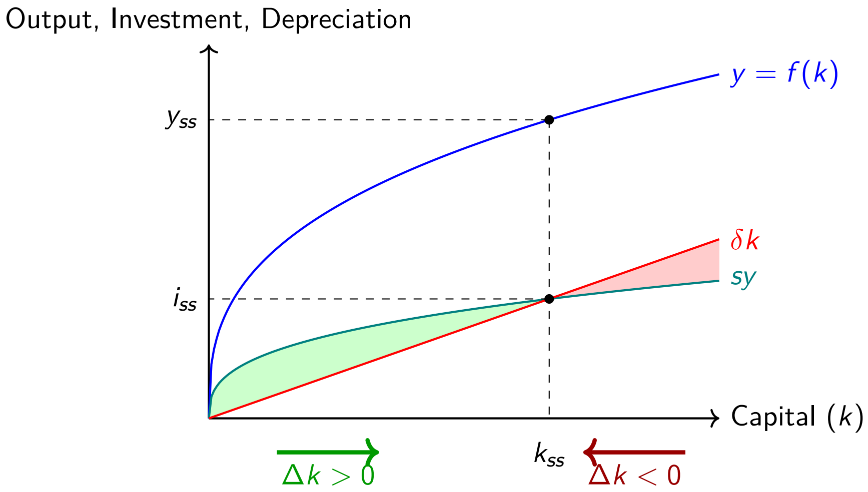
  Depreciation consumes part of the capital stock.

Thus:

$$\Delta k > 0 \iff sf(k) > \delta k$$

$$\Delta k < 0 \iff sf(k) < \delta k$$

Solving the model



The Steady State

The steady state is where capital per worker does not change:

$$k_{t+1} = k_t = k_{ss}$$

Condition:

$$sf(k_{ss}) = \delta k_{ss}$$

- If $k < k_{ss}$: investment exceeds depreciation+dilution $\Rightarrow$ k rises.
- If $k > k_{ss}$: investment is too small to sustain the level of capital $\Rightarrow$ k falls.
- The economy converges to k_{ss} over time.

Closed-Form Steady State: Cobb–Douglas

If:

$$y = Ak^{\alpha}$$

where A denotes a productivity parameter driving higher or lower output given a quantity of capital.

Then, steady state satisfies:

$$sAk_{ss}^{\alpha} = \delta k_{ss}$$

Solving:

$$k_{ss} = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y_{ss} = A\left(\frac{sA}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Why are poor countries poor?

- Now let's use our model to try to rationalize differences in GDP per capita across countries and what could be its main drivers.
- Let's imagine that we have two economies:
  - Poor economy: s^p, A^p
  - Rich economy: s^r, A^r
- Assume that the depreciation rate is equal across these two countries.
- The ratio of GDP per capita across these two economies is given by:

$$\frac{y^r}{y^p} = \left(\frac{A^r}{A^p}\right)^{\frac{1}{1-\alpha}} \left(\frac{s^r}{s^p}\right)^{\frac{\alpha}{1-\alpha}}$$

Why are poor countries poor?

The Role of Investment vs Productivity differences

- With $\alpha = 1/3$, we get:

$$\frac{y^r}{y^p} = \left(\frac{A^r}{A^p}\right)^{3/2} \left(\frac{s^r}{s^p}\right)^{1/2}$$

- The richest countries in the world have a GDP per capita around 70 times larger than poor countries
- Investment rates are around 28%in rich countries and 7% in poor countries
- These means that according to the Solow model, differences in savings rate would make rich countries around $\left(28/7\right)^{1/2} = 2$ times richer
- If poor countries had the productivity of rich countries, they would be 35 times richer! (in contrast to 14 in the previous chapter, why?)

Economic Growth in the Solow Model

Steady State

- The Solow model implies convergence to a **steady state**.
- In the long run:
  - Capital and output are constant: K^*, Y^*.
  - Output per person is constant:

$$y^* \equiv \frac{Y^*}{\bar{L}}$$

  - Consumption per person is constant:

$$c^* = (1 - s)y^*$$

- Long-run growth in y is zero.

Economic Growth in the Solow Model

Transition vs. Data

- The model allows for growth only **during transition**.
- If $K_0 < K^*$:
  - Capital accumulates.
  - Output per person grows temporarily.
- Once $K = K^*$, growth stops.
- But in the data:
  - GDP per capita grows persistently over time.
  - U.S. GDP per capita: $\approx 2\%$ per year for over a century.

Economic Growth in the Solow Model

Key Lesson

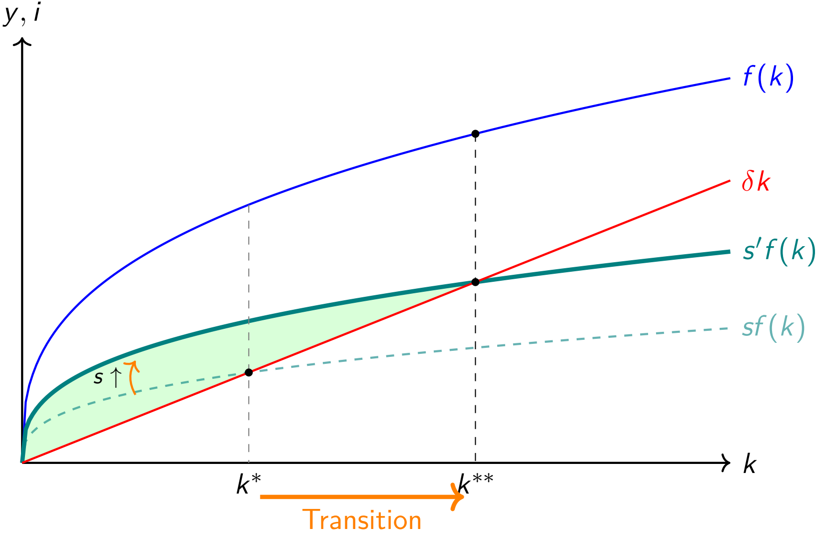
- Capital accumulation cannot explain **long-run** growth.
- Diminishing returns to capital imply:
  - Falling marginal product of capital.
  - Investment eventually only offsets depreciation.
- Physical capital drives **medium-run** growth, not long-run growth.
- Something else must sustain growth.

Increase in the Savings Rate

Suppose an economy in steady state k^* experiences a permanent increase in the savings rate from s to s'.

- The investment curve $sf(k)$ rotates upward to $s'f(k)$. For any given level of capital, the economy now saves and invests more.
- At the old steady state k^*, investment now exceeds depreciation: $s'f(k^*) > \delta k^*$.
- This surplus causes the capital stock to grow ($\Delta k > 0$).
- As k rises, output y also rises along the production function.
- Growth continues but slows down as the economy approaches k^{**}, where $s'f(k^{**}) = \delta k^{**}$.
- The levels of capital per worker and output per worker are permanently higher, but the long-run *growth rate* eventually returns to zero.

Increase in Savings Rate

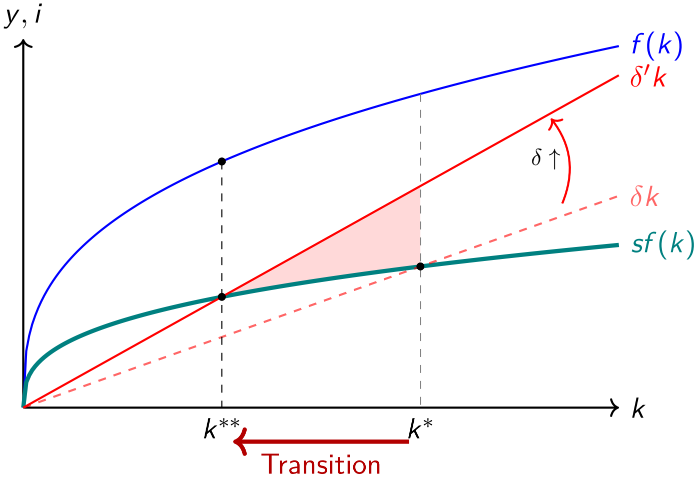


Increase in the Depreciation Rate

Suppose an economy in steady state k^* faces a permanent increase in the depreciation rate from δ to δ'.

- The investment line δk rotates upward (becomes steeper). For every unit of capital, more investment is now required just to maintain the current level.
- At the original steady state k^*, depreciation now exceeds investment: $\delta' k^* > sf(k^*)$.
- The net change in capital becomes negative ($\Delta k < 0$).
- The capital stock begins to decline, which in turn reduces output per worker y along the production function.
- The capital stock continues to contract until it reaches k^{**}, where the lower level of investment is once again sufficient to cover the higher depreciation rate.
- Both capital per worker and output per worker settle at permanently lower levels ($y^{**} < y^*$).

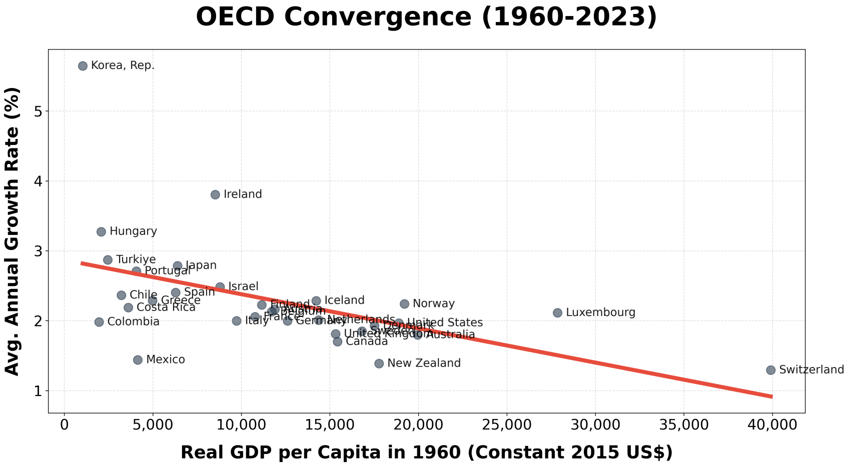
Comparative Statics: Increase in Depreciation



Transitional Dynamics

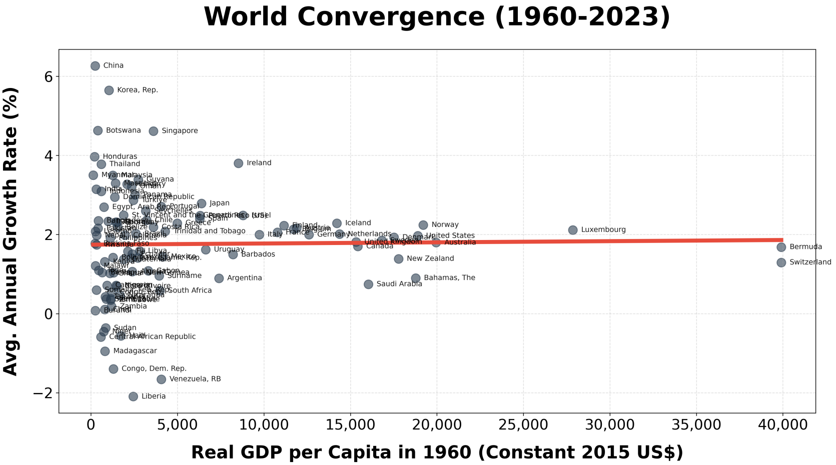
- The Solow model provides a framework for understanding why some countries grow faster than others based on their position relative to their steady state.
- The farther below its steady state an economy is, the faster it will grow.
- As k approaches k_{ss}, the growth rate of output per capita gradually declines toward zero.
- The Solow model predicts **convergence:**
  - Countries with similar steady states should eventually converge
  - Poorer countries should grow faster.
  - China can be viewed as an economy far below its steady state catching up.

Theory vs Data



Source: World Bank WDI. Growth = avg. annual log change.

Theory vs Data



Source: World Bank WDI. Growth = avg. annual log change.

Case Study: Korea vs Afghanistan

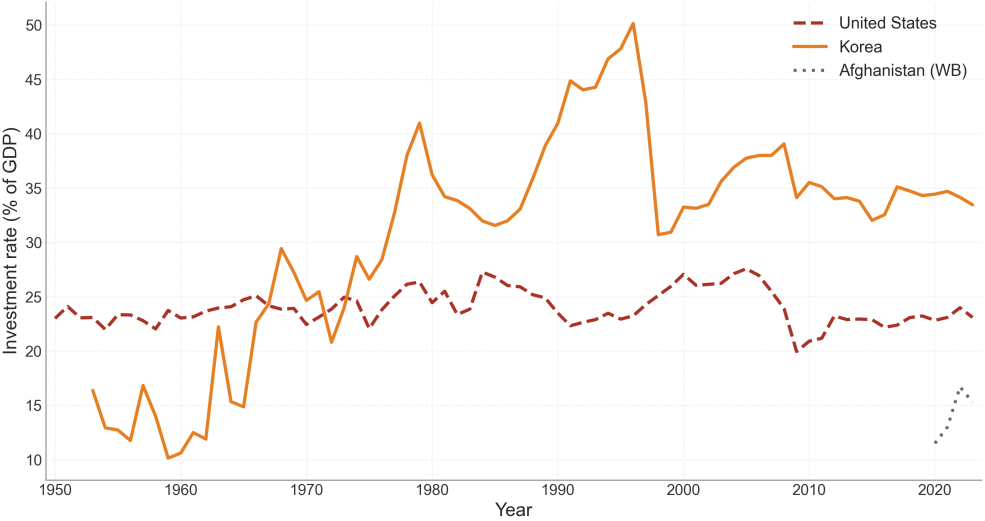
- We started the chapter by asking: Why has South Korea grown so much faster than Afghanistan in the past half century?
- We can apply what we have learned on transitional dynamics to try to rationalize this difference in growth performance.
- We can think that changes in investment and productivity have pushed South Korea to a much higher steady state than Afghanistan.
  - Starting from around 10 percent of U.S. income in 1960, maybe Korea's steady-state level moved up to 50 or 75 percent while the Afghanistan's level remained at 10 percent.
- This would entail that in 1960, Korea was far below its steady state and thus had a lot of room to grow
- Afghanistan was already close to its steady state and thus had little room to grow.

Case Study: Korea vs Afghanistan

- What does the Solow model tell us about the determinants of the steady- state ratio of income in South Korea to income in the United States?
- Assuming these economies have the same rate of depreciation, the ratio of their steady- state incomes is given by:

$$\frac{y^{Korea}}{y^{US}} = \left(\frac{A^{Korea}}{A^{US}}\right)^{3/2} \left(\frac{s^{Korea}}{s^{US}}\right)^{1/2}$$

Investment Rate: US, Korea, and Afghanistan



Source: Penn World Table 11.0 (pwt110.dta). Variable: csh_i (% share of investment in GDP).

Case Study: Korea vs Afghanistan

- The U.S. investment rate is relatively stable
- The investment rate in Korea rises dramatically, from about 15 percent in the 1950s to around 40 percent in the early 1990s.
- According to the Solow model, this rise should significantly increase Korea's steady- state income
- Transitional dynamics tells us that the Korean economy will grow rapidly as it makes the transition to its new, higher level.
- On the other hand, the investment rate in Afghanistan remains at much lower levels for the data that I could find.
- We could think that the steady is trapped at a low level and thus the economy has little room to grow but hopefully the recent increases in investment rate will help the country to grow faster in the future.

Conclusion: What Does the Solow Model Teach Us?

- The Solow framework is a key building block of macroeconomics of growth:
  - A production function with capital and labor.
  - An accumulation equation linking saving today to capital tomorrow.
- **Strengths**:
  - Explains income levels in the long run (investment, TFP, depreciation).
  - Explains growth differences through transition dynamics.
- **Limitations**:
  - Investment differences explain only a small part of income gaps.
  - TFP differences remain largely unexplained.
  - No theory of sustained long-run growth.
- Capital accumulation drives medium-run growth, but long-run growth must come from elsewhere.

Beyond the Solow Model

- The Solow model does not provide a theory of long-run growth.
- Once the economy reaches its steady state, growth stops because of decreasing returns to capital.
- Paul Romer (1990) argued that *ideas* are fundamentally different from capital and can generate increasing returns and therefore a theory of sustained long-run growth.
- Understanding this difference between ideas and capital/labor lets us build a model with endogenous long-run growth.

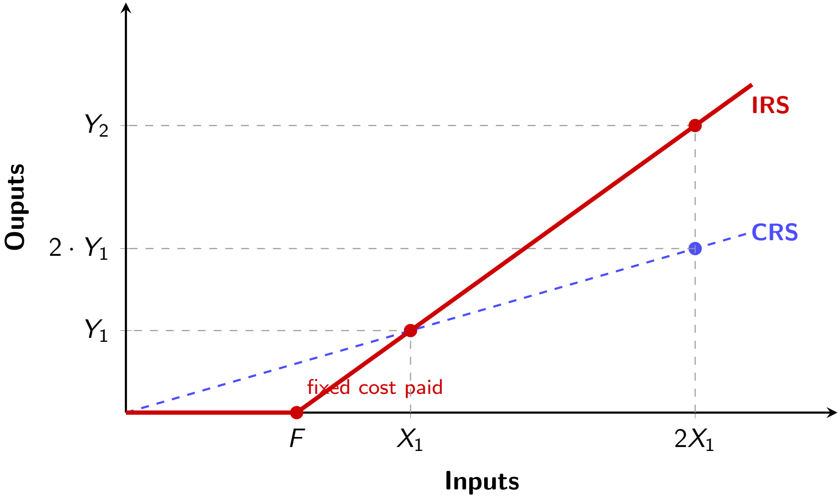
Rival vs. Non-Rival Goods

- **Rival goods**: use by one agent *excludes* use by another.
  - Examples: machines, workers, raw materials.
  - If a firm uses a machine, no other firm can use the same machine simultaneously.
- **Non-rival goods**: use by one agent does *not* prevent use by another.
  - Examples: the recipe for Bolo de Bolacha from your grandmother, blueprints, software, scientific formulas.
  - Newton's laws can be applied by every scientist simultaneously.
- **Ideas are non-rival**: once discovered, they can be used by everyone.

Increasing Returns with Ideas

- Consider the production of a new antibiotic drug:
- The production of the drug requires a fixed cost of $100 million$ to discover the formula, and a variable cost of 1 per unit produced.
- Producing the first unit costs $100 million + 1 = 100,000,001$ dollars.
- Producing the second unit costs only 1 dollar, since the fixed cost has already been paid.
- Producing the *n*-th unit costs only 1 dollar.

Increasing Returns



Increasing Returns with Ideas

- As we have done before, we can think of the production of ideas as a production function:

$$Y_t = F(A_t, L_t, K_t)$$

  where A_t is the stock of ideas.

- Previously, we treated A_t as a fixed constant parameter.

- Given that F(K,L) is constant returns to scale in L and K, F(A,L,K) is increasing returns in L, K, A.

$$F(2A, 2L, 2K) = 2A(2K)^{\alpha}(2L)^{1-\alpha} = 4F(A, L, K) > 2F(A, L, K)$$

Increasing Returns with Ideas

Problems with Perfect Competition

- Under perfect competition, firms should set a price equal to marginal cost.
- In our antibiotic example, the marginal cost of producing an additional unit is 1 dollar.
- If the price is set at 1 dollar, firms will never be able to recover the fixed cost of 100 million dollars.
- This would lead to no production of the drug.
  - Intellectual property rights (patents, copyrights).
  - Government funding for research and development (R&D).
  - Non-market incentives (prizes, recognition).
- Which of these mechanisms—patents, government funding, or prizes—provide the best incentives for innovation and maximize welfare?

The Romer Model: Setup

- The economy has two sectors and a fixed labor supply $\bar{L}$.
- Final goods sector:

$$Y_t = A_t \, L_Y$$

  Output depends on the stock of ideas A_t and labor allocated to production L_Y.
- R&D sector:

$$A_{t+1} - A_t = \delta_A \, A_t \, L_A$$

  The production of new ideas depends on the existing stock of ideas A_t and labor allocated to research L_A, where δ_A is a productivity parameter.
- We assume that a constant fraction ℓ of the labor force is allocated to R&D:
  - $L_Y = (1 - \ell)\bar{L}$
  - $L_A = \ell\bar{L}$
  - Total labor constraint: $L_Y + L_A = \bar{L}$

Solution of the Romer Model

- Output per person:

$$y_t = \frac{Y_t}{\bar{L}} = A_t(1 - \ell)$$

- Growth rate of ideas:

$$g_A = \frac{A_{t+1} - A_t}{A_t} = \delta_A L_A = \delta_A \ell \bar{L}$$

  The growth rate of ideas is constant and depends on the number of researchers nad the productivity of research.

- Ouput per person also grows that the same constant rate.

  $\Rightarrow$ **A Theory of Sustained Growth**

Why there is sustained growth in the Romer model?

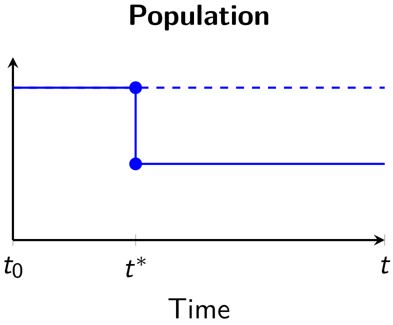
- In Solow, the accumulation of capital runs into diminishing returns: each additional unit of capital adds less and less to output.
- Eventually, investment only offsets depreciation and growth stops.
- In Romer, the accumulation of ideas does not run into diminishing returns:

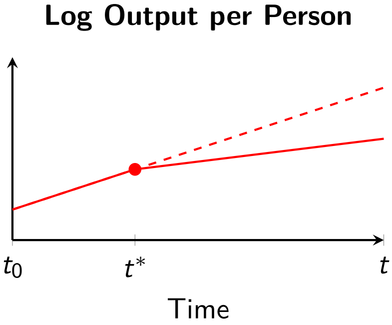
$$A_{t+1} - A_t = \delta_A A_t L_A$$

- As we accumulate more knowledge, the return to knowledge does not fall.
- Old ideas continue to help us produce new ideas in a virtuous circle that sustains economic growth.

Lower Population

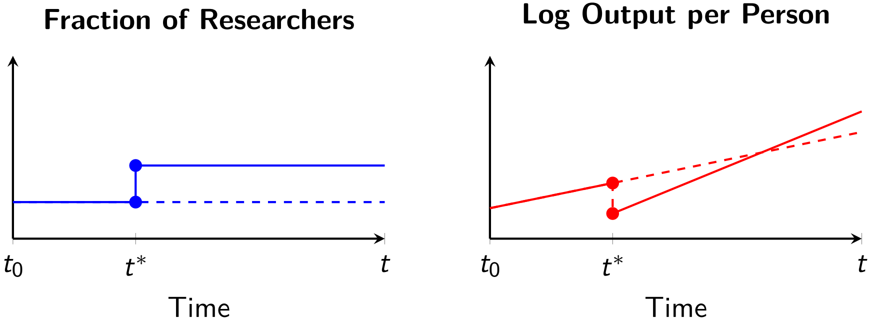
- A decrease in population $\bar{L} \to \tilde{L}$ decreases the number of researchers $L_A = \ell \bar{L}$, which slows down the growth rate of ideas $\tilde{g}_A = \delta_A \ell \tilde{L}$.
- This is the **scale effect**: larger economies grow faster because they have more researchers.





Higher Fraction of Researchers

- An increase in the fraction of researchers $\ell \to \tilde{\ell}$ increases the growth rate of ideas $\tilde{g}_A = \delta_A \tilde{\ell} \bar{L}$ and output.
- But as people are allocated to research, less people are allocated to production, so the level of output per person falls.
- Increasing the research share involves a trade-off.



Conclusion

- The Solow model explains medium-run growth through capital accumulation, but long-run growth must come from elsewhere.
- The Romer model provides a theory of sustained long-run growth.
- Because ideas are nonrivalrous, it is not ideas per person that matter, but rather this total stock of ideas.
- Increases in the stock of knowledge lead to sustained economic growth for countries that have access to that knowledge.