Money, Inflation, and Output

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Exercise 1. Thieves Steal Half the Gold in Lisbon

Consider the simple monetary model of Lisbon in the 1500s. The quantity equation is

$$M_t V = P_t Y_t,$$

where velocity is constant. Firms set prices one period in advance and update them according to

$$P_{t+1} = P_t \left(\frac{Y_t}{Y^*} \right)^{\theta}, \qquad 0 < \theta < 1.$$

Equivalently, inflation between periods t and $t+1$ is

$$\pi_{t+1} \equiv \ln P_{t+1} - \ln P_t = \theta \ln \left(\frac{Y_t}{Y^*} \right).$$

Suppose the normal money supply is $\bar{M}$, and the normal price level is

$$\bar{P} = \frac{\bar{M} V}{Y^*}.$$

Before the episode begins, Lisbon has received a very large amount of gold from abroad, so the money supply is

$$M_0^{\text{before theft}} = 4\bar{M}.$$

Prices have not yet adjusted, so the price level at the start of period 0 is still

$$P_0 = \bar{P}.$$

At the start of period 0, a group of thieves steals half of all the gold in Lisbon and takes it out of circulation. After the theft,

$$M_0 = 2\bar{M}.$$

No more gold is stolen or discovered after period 0, so $M_t = 2\bar{M}$ for all $t \geq 0$.

(a) Using the quantity equation, compute output in period 0 after the theft. Express your answer as a ratio Y_0/Y^*.

(b) Is output above or below desired output in period 0? Explain why the theft reduces the boom but does not eliminate it.

(c) Compute inflation between period 0 and period 1.

(d) Find the new long-run steady-state price level $\hat{P}$ after the theft. Compare it with the original normal price level $\bar{P}$.

(e) Suppose $\theta = 1/2$. Compute $P_1/\bar{P}$ and Y_1/Y^*.

(f) Show that if $P_t < \hat{P}$, then $Y_t > Y^*$ and inflation is positive. Explain the transition of prices and output from period 0 to the new steady state.

(g) How would your answer change if the thieves had stolen enough gold to leave only $M_0 = 0.5\bar{M}$ in circulation? Would inflation still be positive?

Exercise 2. The King Debases Gold Coins to Pay His Debt

Consider again the same model:

$$M_t V = P_t Y_t, \qquad P_{t+1} = P_t \left(\frac{Y_t}{Y^*} \right)^{\theta}, \qquad 0 < \theta < 1.$$

Lisbon starts in its normal steady state:

$$M_{-1} = \bar{M}, \qquad P_0 = \bar{P} = \frac{\bar{M}V}{Y^*}, \qquad Y_{-1} = Y^*.$$

At the start of period 0, King Joao III faces a large debt. Instead of raising taxes, he orders the mint to melt down the existing gold coins and issue lighter coins with less gold content. The same amount of gold is transformed into more coins. The number of coins in circulation rises by 50 percent:

$$M_0 = 1.5\bar{M}.$$

From period 1 onward, the king keeps debasing the currency at a constant rate $\mu > 0$:

$$M_{t+1} = (1 + \mu) M_t.$$

(a) Explain why this policy is a form of seigniorage. Who pays the inflation tax?

(b) Prices were chosen before the debasement was announced, so $P_0 = \bar{P}$. Use the quantity equation to compute Y_0/Y^*.

(c) Compute inflation between period 0 and period 1.

(d) Suppose $\theta = 0.5$ and $\mu = 0.10$. Compute π_1. In the long run, what is the inflation rate?

(e) Suppose the king wants to guarantee positive inflation every period. Explain why he must keep increasing the money supply over time, instead of debasing the currency only once.