Work and Leisure

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Exercise 1. Income and Substitution Effects in Labor Supply

Consider a household with utility

$$u(c, \ell) = \ln c + \phi \ln \ell, \qquad \phi > 0,$$

where c is consumption and ℓ is leisure. Time endowment is 1, so hours worked are

$$N = 1 - \ell.$$

The household faces the budget constraint

$$c = w(1 - \ell) + T,$$

where $w > 0$ is the wage and $T \geq 0$ is a lump-sum transfer.

(a) Write the household's optimization problem in terms of leisure only. Derive the first-order condition.

(b) Solve for optimal leisure $\ell^*(w, T)$ and labor supply $N^*(w, T)$.

(c) Show that an increase in the transfer T reduces labor supply. Why is this a pure income effect?

(d) Suppose $\phi = 1$.

- Compute optimal leisure, labor supply, and consumption when $w = 8$ and $T = 0$.
- Compute them again when $w = 8$ and $T = 2$.

(e) Using your formula for $N^*(w, T)$, study the effect of an increase in the wage:

- first when $T = 0$,
- then when $T > 0$.

In each case, explain the relative importance of the income and substitution effects.

(f) In the slides, we saw that labor supply can be upward sloping or downward sloping. Does this utility specification generate a backward-bending labor supply curve? Explain.

Exercise 2. General Equilibrium with Endogenous Labor and an AI Shock

Consider a one-period competitive economy. A representative household chooses consumption c and hours worked N to maximize

$$u(c, N) = c - \frac{\psi}{2}N^2, \qquad \psi > 0.$$

The household owns the fixed capital stock $\bar{K}$ and receives both labor income and capital income. The firm's production function is

$$Y = A(z\bar{K})^{\alpha}N^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where $A > 0$ is TFP and $z > 0$ measures the productivity of capital. You can interpret an increase in z as an AI shock that makes capital more effective.

Markets are competitive, so factor prices satisfy

$$w = \frac{\partial Y}{\partial N}, \qquad r = \frac{\partial Y}{\partial \bar{K}}.$$

(a) Write down the household budget constraint.

(b) Derive the household first-order condition for labor supply and show that

$$N = \frac{w}{\psi}.$$

Explain how changes in the wage w affect labor supply and wether the income or substitution effect dominates.

(c) Derive the firm's labor demand and capital demand equations:

$$w = (1-\alpha)A(z\bar{K})^{\alpha}N^{-\alpha}, \qquad r = \alpha A z^{\alpha}\bar{K}^{\alpha-1}N^{1-\alpha}.$$

(d) Write the full general-equilibrium system of equations. List the endogenous and exogenous variables.

(e) Use the household labor-supply condition and the firm's labor-demand condition to solve for equilibrium labor N^*. Show that labor is determined *inside* the model, unlike in the production model from Chapter II where labor was fixed exogenously at $\bar{L}$.

Hint: combine

$$w = \psi N \qquad \text{and} \qquad w = (1-\alpha)A(z\bar{K})^{\alpha}N^{-\alpha}$$

to obtain one equation in N only.

(f) **AI shock.** Suppose z increases permanently.

- What happens to equilibrium output?
- What happens to the equilibrium wage?
- What happens to the rental rate of capital?
- What happens to equilibrium labor supply in this model?