Macroeconomics I
Homework 1

Due to Wednesday the 20th of November at 11 a.m.

Exercise 1

Consider a simple exchange economy with two consumers, indexed by $i = 1, 2$, who live forever, and with one perishable consumption good. Time is discrete and indexed by $t = 0, 1, \ldots \ldots$ Each consumer values sequences of consumption goods, $c^i = \left\{c_t^i\right\}_{t=1}^{\infty}$ according to

$$U\left(c^i\right) = \sum_{t=0}^{\infty} \beta^t \ln\left(c_t^i\right)$$

with $\beta \in (0, 1)$. The endowment processes are given by

$$w_t^i = \begin{cases} w_t^1 = 2 \text{ and } w_t^2 = 0 \text{ if } t \text{ is even or } 0 \\ w_t^1 = 0 \text{ and } w_t^2 = 2 \text{ if } t \text{ is odd} \end{cases}$$

a Define a Arrow-Debreu equilibrium for this economy. What are the price dynamics? Which agent consumes more?

b Characterize the set of Pareto optimal allocations.

c Find transfers as a function of α that decentralize the Pareto optimal allocation as a a competitive allocation.

d Using the programming language of your choice, build a function that takes as argument the value of α_1 and computes the transfer that the social planner would give to agent 1 in order to decentralize the equilibrium. Then, construct a grid of α_1 with 100 points between $[0, 1]$ and plot how transfers change with α_1.

Exercise 2

Consider a simple exchange economy with two consumers, indexed by $i = 1, 2$, who live forever and instant utility given by:

$$u^i(c_t^i) = \log(c_t^i) \text{ for } i = 1, 2$$

Suppose there are two states, $S = \{1, 2\}$. Let $s_0 = 1$, and

$$\text{prob}\left(s_{t+1} = 1 | s_t = 1\right) = \pi_{11}, \text{ and } \text{prob}\left(s_{t+1} = 2 | s_t = 1\right) = \pi_{12} = 1 - \pi_{11}$$

Transition probabilities π_{22} and π_{21} are defined similarly. The endowment functions are

$$w_t^1 = s_t$$

and

$$w_t^2 = 3 - s_t$$

a Determine $p_t(s^t)$ (the price of a good in history s^t in an Arrow-Debreu market structure) as a function of π and β. Set $p_0(s_0)$ as the numeraire.

b Now imagine that $\pi_{11} = \pi_{22} = 0.5$. Define the consumption value of each agent through time.

c Using the programming language of your choice, simulate a history of s_t for 20 periods with $\pi_{11} = \pi_{22} = 0.5$. For the particular history of shocks that you have simulated, plot c_t^1 and $c_t^1 - w_t^1$ from $t = 1$ to $t = 20$.