Huggett (JME, 1996)

In this exercise, I give you a cook book procedure for solving Huggett (1996). I encourage you to work in Fortran, Julia, or C++. You can work in groups of 2/3 people. I expect each group to present a small report (no code) on the session of the 18th of November (points 1 to 9) and on the 25th of November (points 10 to 12).

1. Build an equidistant grid of capital $a \in (0, a_{\max})$, you can start with a value of $a_{\max} = 100$, then we will check if we need to expand or shrink the grid and a number of points $nkk = 100$.

2. Discretize the income shock z and compute the transition probability matrix using the Tauchen method.

$$\log z_t = \rho \log z_{t-1} + \epsilon_t, \epsilon \sim N(0, \sigma_\epsilon)$$

   with $\rho = 0.90$ and $\sigma_\epsilon = 0.2$. Normalize the labor market productivity vector such that the unconditional mean of the shock z equals 1 (the level, not the log, of the shock needs to have mean one).

3. Load the survival rate probabilities provided in the txt file (LifeTables.txt) and compute the fraction of population ψ_j in each age group j.

4. The function describing the efficiency units of labor is given by:

$$e(j,z) = \begin{cases} zP(j), j < J_R \\ 0, j \geq J_R \end{cases},$$

   and the age polynomial is given by,

$$P(j) = \lambda_0 + \lambda_1 j + \lambda_2 j^2,$$

   with $\lambda_0 = 0.195$, $\lambda_1 = 0.107$, and $\lambda_2 = -0.00213$. Individuals enter the model in period 1 (corresponding to real age 25) retire in period $J_R = 41$ (corresponding to real age 65), and die for sure in period $J = 71$ (corresponding to real life age of 95). The rate of growth of the size of new cohorts n is set to 1 percent. Compute total labor supply L.

5. Assume the following parameters:
   (a) $\sigma = 2$, $\beta = 0.96$, $\underline{a} = 0$
   (b) Cobb-Douglas production function: capital share=0.36, depreciation rate=0.08, aggregate productivity =1.

6. Set an interest rate $r^g = 0.02$.

7. For the guess of the interest rate and given L compute:
   (a) Using firms FOCs: aggregate capital demand and wages.
   (b) Equilibrium payroll tax θ and associated pension b given a replacement rate $\omega = 0.5$

8. Make a guess of accidental bequests $T^g = 1.2$.

9. Given all parameters, prices and transfers solve for the household problem to obtain the policy function $g_j^a(z, a)\ \forall j \in [0, J]$

10. Given the $g_j^a(z, a)$ simulate a panel of artificial individuals from $j = 1$ to $j = J$. In period 0, set individuals assets to zero and sample the income shock from the unconditional distribution of z. Based on the simulation compute aggregate transfers T^{g+1} and capital supply from households.

11. If $T^{g+1} = T^g$ continue, otherwise set $T^g = T^{g+1}$ and move back to 9.

12. Compute the interest rate r^{g+1} associated with capital supplied by households. If $r^{g+1} = r_g$ you are done, otherwise set $r^g = 0.5r_g + 0.5r^{g+1}$ and go back to 7 until convergence.

The report to be presented on the 25th of November should provide:

a Plot of policy function $a' - a$ against a for two different ages (before and after retirement).

b Plot the euler equation error given the linear approximation of the policy function for a very fine grid (10,000 points).

The report to be presented on the 15th of February should provide:

a Equilibrium interest rate

b Equilibrium wage

c Income tax

d Pension

e Capital to output ratio

f Gini index

g Histogram of asset holdings

h The mean, the median, the top and bottom quartiles of the asset distribution for each age.

i Average and Gini index of earnings for each age.

j Average and Gini index of consumption for each age.