Exercise 1

Each one of you should pick one of the following functions:

$$f_1(x) = -13(x+1)^{-3} + 13 \quad x \in [0,5]$$
$$f_2(x) = \begin{cases} 10x & x \in [0,1] \\ 2\log(x) + 10 & x \in (1,5] \end{cases}$$
$$f_3(x) = \begin{cases} 10x & x \in [0,1] \\ 10 & x \in (1,5] \end{cases}$$
$$f_4(x) = \frac{1}{x + \frac{1}{13}} \quad x \in [0,5]$$
$$f_5(x) = \frac{4}{5}\left(\frac{1}{x + \frac{1}{13}}\right) + \frac{1}{5}\left(13 - \frac{13}{5}x\right) \quad x \in [0,5]$$

Using a finite number of elements p two different basis (monomials and Chebyshev polynomials) $\Psi = \{\psi_i(\cdot)\}_{i=0}^{\infty}$ of the space of continuous functions approximate the function using Dirac delta, least squares, and Galerkin projection.

Exercise 2

An agent solves the following problem:

$$\max_{a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta s_j \mathbf{E}[\frac{c^{1-\sigma}}{1-\sigma}]$$
$$\text{s.t. } c + a' = a$$
$$c' = (1+r)a' + \exp(\epsilon),$$

where $\epsilon \sim N(0, \sigma_\epsilon)$.

Solve the model using:

1. Collocation/ Dirac Delta projection
2. Least squares projection
3. Galerkin projection

Plot the policy function associated to each projection method and the log of the absolute value of the residual of the euler equation in an interval of $a \in [a_{min}; a_{max}]$

Use the following parameter values:

$$\sigma = 2 \qquad \sigma_\epsilon = 0.2$$
$$a_{\min} = 1 \qquad a_{\max} = 20$$
$$\beta = 0.95 \qquad r = 0.05$$
$$s_j = 0.9$$